

ANALYSIS OF FUNCTIONS/TRIG **SUMMER PACKET**

Directions: This packet is required if you are registered for Precalculus for the upcoming school year. The packet will be collected on the first day of school and given homework points. The topics in this packet are all skills that you have already seen, and it is necessary that you are comfortable working problems of these types upon entering **AO/TRIG**. Use the provided example problems and your notes from previous classes to assist in completion.

Functions**Operations with Functions**

Example: Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, $\left(\frac{f}{g}\right)(x)$ for: $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$.

$$\begin{aligned}(f + g)(x) &= f(x) + g(x) \\ &= (x^2 + 3x - 4) + (3x - 2) \\ &= x^2 + 6x - 6\end{aligned}$$

$$\begin{aligned}(f - g)(x) &= f(x) - g(x) \\ &= (x^2 + 3x - 4) - (3x - 2) \\ &= x^2 + 3x - 4 - 3x + 2 \\ &= x^2 - 2\end{aligned}$$

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x) \\ &= (x^2 + 3x - 4)(3x - 2) \\ &= x^2(3x - 2) + 3x(3x - 2) - 4(3x - 2) \\ &= 3x^3 - 2x^2 + 9x^2 - 6x - 12x + 8 \\ &= 3x^3 + 7x^2 - 18x + 8\end{aligned}$$

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{x^2 + 3x - 4}{3x - 2}, x \neq \frac{2}{3}\end{aligned}$$

Practice: Perform the following operations for: $f(x) = 2x + 1$ and $g(x) = x - 3$.

1. $(f + g)(x)$

2. $(g - f)(x)$

3. $(f - g)(x)$

4. $(f \cdot g)(x)$

5. $\left(\frac{g}{f}\right)(x)$

6. $\left(\frac{f}{g}\right)(x)$

Evaluating Functions

Examples: $m(x) = 2x^2 - x + 2$

$$\begin{aligned}m(2) &= 2(2)^2 - (2) + 2 \\ &= 8\end{aligned}$$

$r(x) = -x^3 + x + 1$

$$\begin{aligned}r(-4) &= -(-4)^3 + (-4) + 1 \\ &= 61\end{aligned}$$

Practice: Evaluate for: $c(x) = -16x^2 - 4x + 2$, $h(x) = x^4 - 2x$, $k(x) = 2x^3 + 5x^2$.

7. $h(-2)$

8. $k(3)$

9. $c\left(\frac{1}{2}\right)$

10. What does $f(2)=5$ mean?

Factoring

Examples: Factor.

GCF:

$$4x^3y + 6x^2y - 2x$$

$$= 2x(2x^2y + 3xy - 1)$$

Difference of 2 Squares:

$$4x^2 - 9$$

$$= (2x)^2 - (3)^2$$

$$= (2x - 3)(2x + 3)$$

Product/Sum:

(or reverse FOIL)

$$x^2 + 2x - 15$$

$$= (x + 5)(x - 3)$$

x (-15)	+ (2)
-1, 15	14
1, -15	-14
-5, 3	-2
5, -3	+2



By Grouping:

$$x^3 - 4x^2 + 2x - 8$$

$$= (x^3 - 4x^2) + (2x - 8)$$

$$= x^2(x - 4) + 2(x - 4)$$

$$= (x - 4)(x^2 + 2)$$

- Use ()+() to group
- Factor out the GCF of each group
- Factor out the common () out of each term

Master Product:

(or just guess and check)

$$2x^2 - 7x + 3$$

x 6	+ (-7)
2, 3	5
-2, -3	-5
1, 6	7
-1, -6	-7

$$= 2x^2 - \overbrace{1x - 6x}^{-7x} + 3$$

$$= (2x^2 - 1x) + (-6x + 3)$$

$$= x(2x - 1) - 3(2x - 1)$$

$$= (2x - 1)(x - 3)$$

- In $ax^2 + bx + c$ form, multiply $a \times c$
- Find 2 #'s that multiply to axc and add to b
- Replace the middle term
- Factor by grouping.

Practice: Factor the following completely.

1. $5a^2b + 10ab^3$

2. $x^2 - 25$

3. $x^2 + 6x$

4. $1 - 9x^2$

5. $6x^3 - 9x^2 + 2x - 3$

6. $5x^2 - 20$

7. $x^2 - 8x + 15$

8. $5x^2 - 7x + 2$

9. $2x^2 - 2x - 24$

10. $6x^2 - x - 1$

Solving Quadratic Equations

Examples: Solve the following quadratic equations.

a.) Solve by factoring:

$$\begin{aligned}x^2 + 2x - 15 &= 0 \\(x - 3)(x + 5) &= 0 \\x - 3 = 0 \quad x + 5 = 0 \\ \boxed{x = 3 \quad x = -5}\end{aligned}$$

- Factor the equation
- Set each factor = to 0.
- Solve for x.

b.) Solve by using the quadratic equation:

$$\begin{aligned}2x^2 - 3x - 6 &= 3 \\2x^2 - 3x - 9 &= 0 \quad a = 2, b = -3, c = -9 \\x &= \frac{+3 \pm \sqrt{(-3)^2 - 4(2)(-9)}}{2(2)} \\x &= \frac{3 \pm \sqrt{81}}{4} \\x = \frac{3+9}{4} = \frac{12}{4} = 3 \quad \swarrow \quad \searrow \quad x = \frac{3-9}{4} = \frac{-6}{4} = \frac{-3}{2} \quad \boxed{x = 3, \frac{-3}{2}}\end{aligned}$$

- Set up in $ax^2 + bx + c = 0$ form. and determine a,b,c
- Substitute into the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Simplify.

c.) Solve by using the quadratic equation, simplify the radicals:

$$\begin{aligned}x^2 - 4x &= 8 \\x^2 - 4x - 8 &= 0 \quad a = 1, b = -4, c = -8 \\x &= \frac{+4 \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2(1)} \\x = \frac{4 \pm \sqrt{48}}{2} &\longrightarrow x = \frac{4 \pm 4\sqrt{3}}{2} \longrightarrow \boxed{x = 2 \pm 2\sqrt{3}}\end{aligned}$$

Practice: Solve.

1. $x^2 + 7x + 12 = 0$

2. $5x^2 = 10x$

3. $2x^2 + x = 15$

4. $-x^2 + 2x + 10 = 0$

5. $x^2 - 16 = 0$

6. $2x^2 - 3x - 3 = 0$

Shifting Graphs

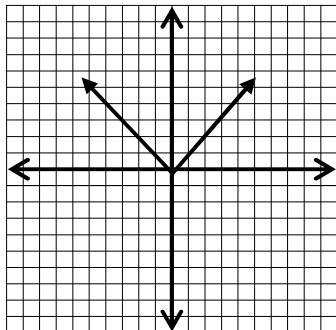
Example:

Be familiar with the graphs of common functions.

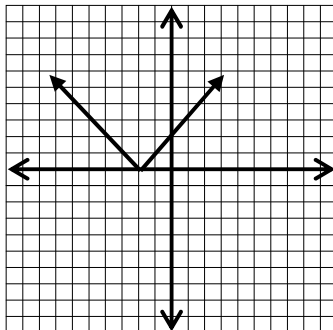
Clearly shift 3 points.

- Vertical shift c units upward: $h(x)=f(x)+c$
- Vertical shift c units downward: $h(x)=f(x)-c$
- Horizontal shift c units to the right: $h(x)=f(x-c)$
- Horizontal shift c units to the left: $h(x)=f(x+c)$

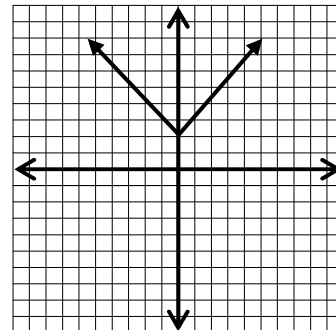
$$f(x) = |x|$$



$$f(x) = |x+2|$$

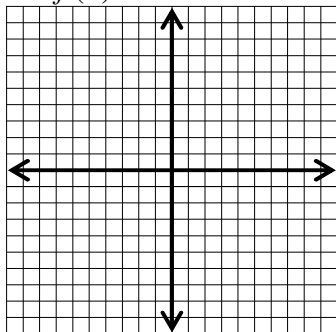


$$f(x) = |x|+2$$

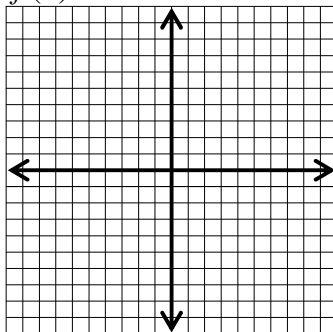


Practice:

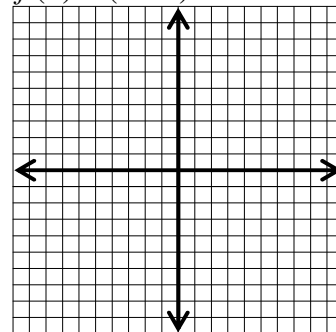
1. $f(x) = x^2$



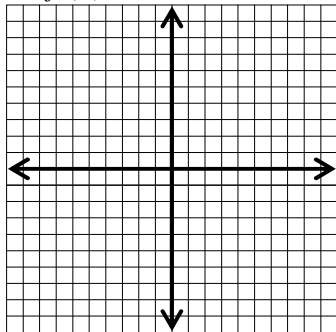
$$f(x) = x^2 - 3$$



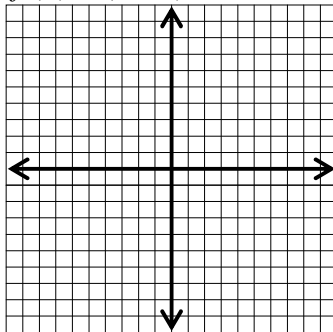
$$f(x) = (x-2)^2 + 1$$



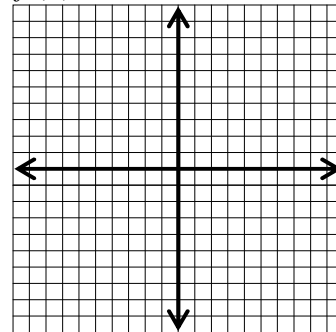
2. $f(x) = x^3$



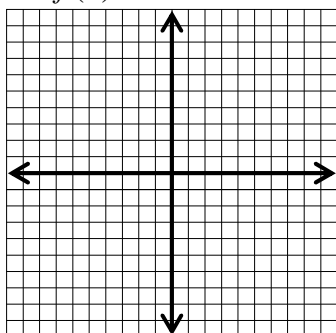
$$f(x) = (x+4)^3$$



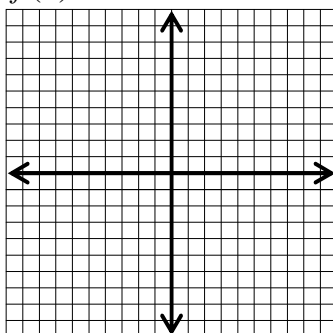
$$f(x) = x^3 - 2$$



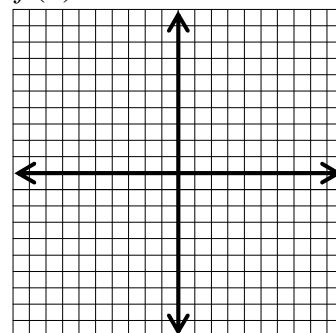
3. $f(x) = \sqrt{x}$



$$f(x) = \sqrt{x} + 4$$



$$f(x) = \sqrt{x+3}$$



Piecewise functions: Graphing and Evaluating

Piecewise functions are composed of two or more functions.

Given the function: $f(x) = \begin{cases} 2x + 3, & x \leq 1 \\ -x + 4, & x > 1 \end{cases}$

To the left of $x = 1$, the graph is the line $y = 2x + 3$.

To the right of $x = 1$, the graph is the line $y = -x + 4$

Evaluate:

$$\begin{aligned} f(-3) &= 2(-3) + 3 = -3 \\ f(-2) &= 2(-2) + 3 = -1 \\ f(-1) &= 2(-1) + 3 = 1 \\ f(0) &= 2(0) + 3 = 3 \\ f(1) &= 2(1) + 3 = 5 \end{aligned}$$

$$\begin{aligned} f(2) &= -2 + 4 = 2 \\ f(3) &= -3 + 4 = 1 \\ f(4) &= -4 + 4 = 0 \\ f(5) &= -5 + 4 = -1 \end{aligned}$$

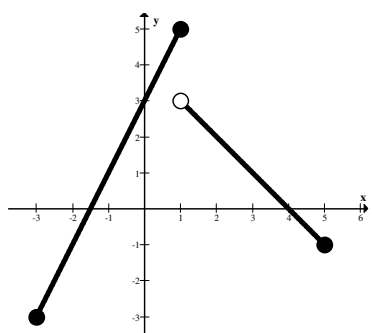


Use the first piece



Use the second piece

To graph, use the points from above



x	y
-3	-3
-2	-1
-1	1
0	3
1	5

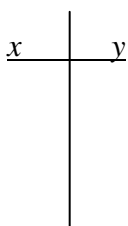
x	y
2	2
3	1
4	0
5	-1
1	3

****Open Dot****

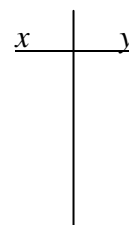
**** You must evaluate the break point in each piece! One point will be a closed dot and the other will be an open dot,**

Evaluate and graph.

1. $f(x) = \begin{cases} 3x + 3, & x < 0 \\ 5 - x, & x \geq 0 \end{cases}$



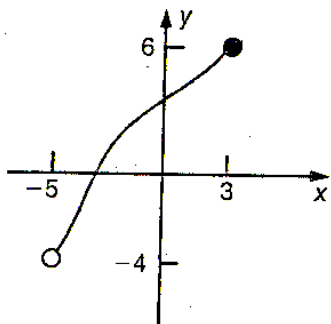
2. $f(x) = \begin{cases} x + 1, & x \leq -1 \\ 2x + 2, & x > -1 \end{cases}$



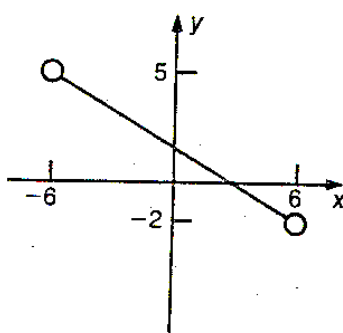
Domain and Range Continued

Identifying the domain and range from the graph is easy. For domain values, take the x-values from left to right. For range values, take the y-values from bottom to top.

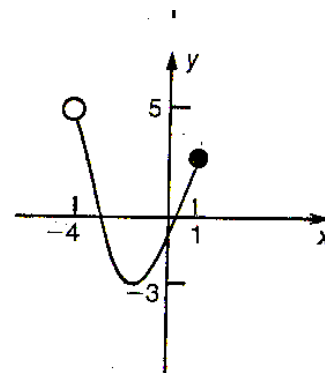
1. Domain: $-5 < x \leq 3$
 Range: $-4 < y \leq 6$



Domain: $-4 < x \leq 1$
 Range: $-3 \leq y < 5$



Domain: $-6 < x < 6$
 Range: $-2 < y < 5$



Practice: Identify the domain for each function.

1. $f(x) = \frac{3x}{x+2}$

2. $g(x) = \frac{x-1}{3x+2}$

3. $h(x) = \sqrt{2x-4}$

4. $f(x) = x^2 + 3x - 4$

5. $g(x) = \sqrt{5-x}$

6. $h(x) = \frac{3}{1+x^2}$

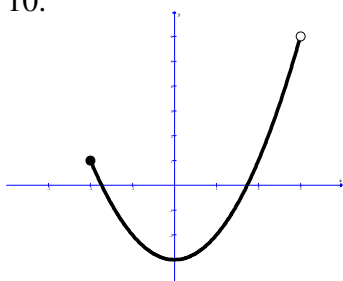
7. $f(x) = \frac{3}{x^2 - x - 6}$

8. $g(x) = \sqrt{x^2 + 9x + 20}$

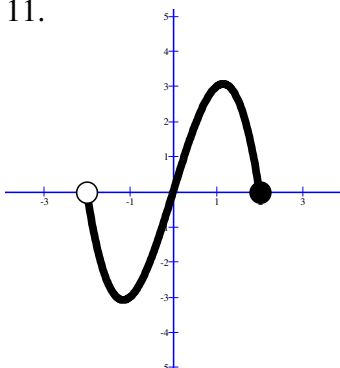
9. $h(x) = \frac{x}{\sqrt{x^2 - 4}}$

Practice: Identify the domain and range for each function.

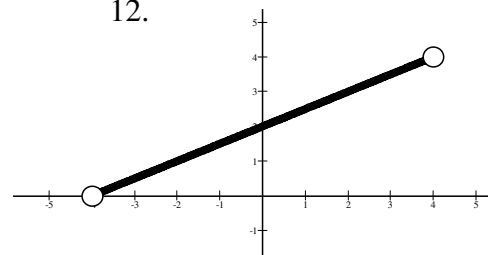
10.



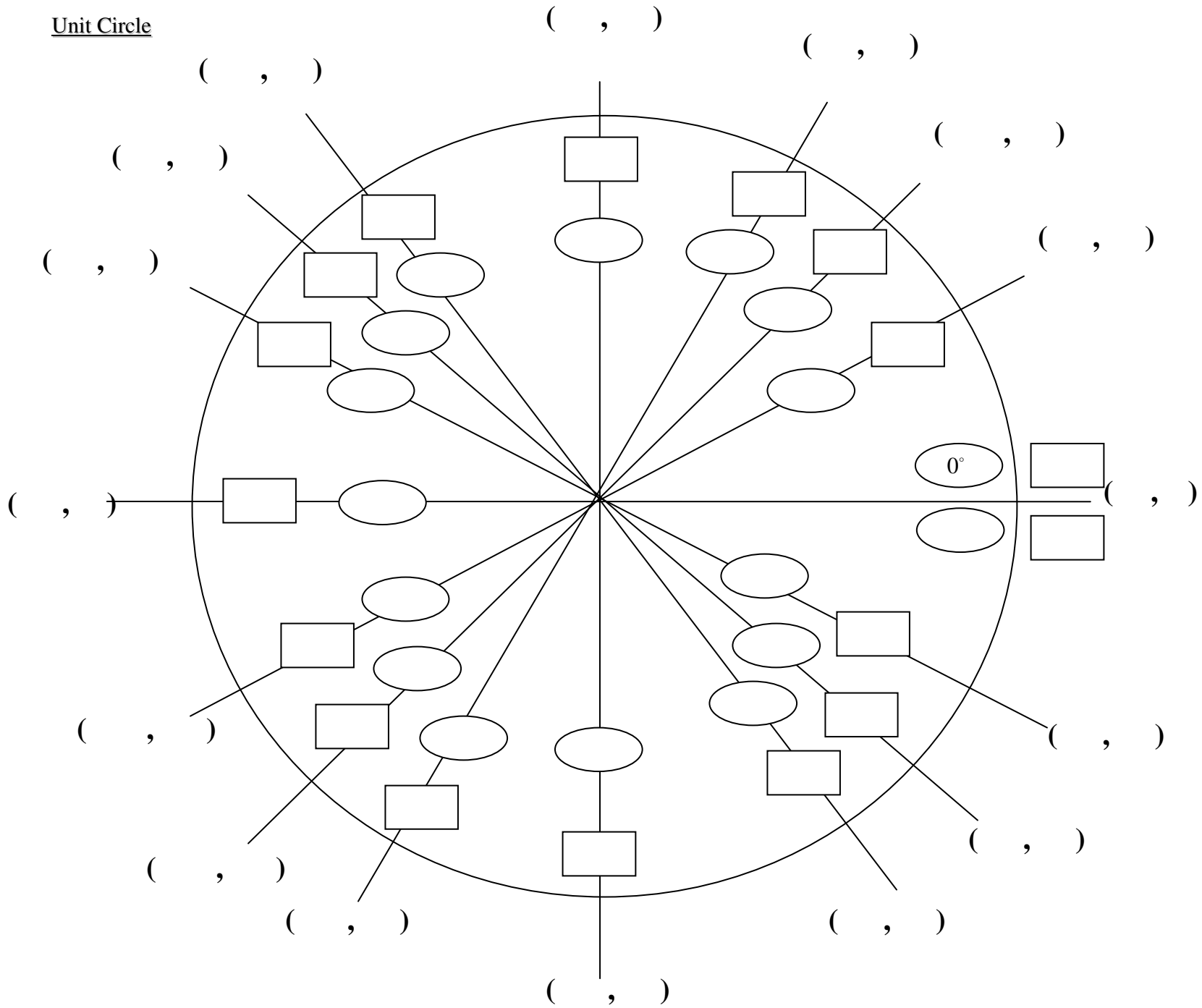
11.



12.



Unit Circle



Key

(x, y)

↓
 $\cos \theta, \sin \theta$

○

↓
Degrees

□

↓
Radians